Unit 6, Lesson 2
Truth and Equations

Goals

- Comprehend the word “variable” to refer to a letter standing in for a number and recognize that a coefficient next to a variable indicates multiplication (in spoken and written language).
- Generate values that make an equation true or false and justify (orally and in writing) whether they are “solutions” to the equation.
- Use substitution to determine whether a given number makes an equation true.

Required Materials

- Cool-down

2.1 Three Letters

Warm-up: 10 minutes

2 minutes of quiet work time for the first part of the first question. Explain the meaning of variable and demonstrate why $a + b = c$ is false for 3, 4 and 5. 2 minutes to complete the rest of the task, followed by a whole-class discussion.

Student-Facing Task Statement

1. The equation $a + b = c$ could be true or false.
   a. If a is 3, b is 4, and c is 5, is the equation true or false?
   b. Find new values of a, b, and c that make the equation true.

Possible Responses

Answers vary. Sample responses:

1. a. False
   b. a is 1, b is 2, c is 3

c. Find new values of a, b, and c that make the equation false.

2. The equation $x \cdot y = z$ could be true or false.
   a. If x is 3, y is 4, and z is 12, is the equation true or false?
   b. Find new values of x, y, and z that make the equation true.
   c. Find new values of x, y, and z that make the equation false.

c. a is 4, b is 5, c is 10

2. a. True
   b. x is 3, y is 5, z is 15
   c. x is 1, y is 2, z is 3

2.2 Storytime

Classroom activity: 15 minutes

Introduce the "next to" notation. Explain that $20x$ means the same thing as $20 \cdot x$, and that the 20 in $20x$ is called the "coefficient." 5 minutes of quiet work time, followed by a whole-class discussion.

Student-Facing Task Statement

Here are three situations and six equations. Which equation best represents each situation? If you get stuck, draw a diagram.

1. After Elena ran 5 miles on Friday, she had run a total of 20 miles for the week. She ran x miles before Friday.
2. Andre's school has 20 clubs, which is five times as many as his cousin's school. His cousin's school has x clubs.

Possible Responses

1. $x + 5 = 20$
2. $5x = 20$
3. $20x = 5$

Anticipated Misconceptions

Students who focus on key words might be misled in each situation. For the first situation, students

3. Jada volunteers at the animal shelter. She divided 5 cups of cat food equally to feed 20 cats. Each cat received x cups of food.

$x+5=20$	$x=20+5$	$5x=20$
$x+20=5$	$5 \cdot 20=x$	$20x=5$

might see the word “total” and decide they need to add 5 and 20. In the second situation, the words “five times as many” might prompt them to multiply 5 by 20. The third story poses some additional challenges: students see the word “divided” but there is no equation with division. Additionally, students might think that division always means divide the larger number by the smaller. Here are some ways to help students make sense of the situations and how equations can represent them:

- Suggest that students act out the situation or draw a picture. Focus on what quantity in the story each number or variable represents, and on the relationships among them.
- Use tape diagrams to represent quantities and think about where a situation describes a total and where it describes parts of the total.
- Ask students about the relationships between operations. For the third situation, ask what operation is related to dividing and might help describe the situation.

2.3 Using Structure to Find Solutions

Classroom activity: 15 minutes

Groups of 2. 10 minutes to work quietly and then share responses with a partner, followed by a whole-class discussion.

Student-Facing Task Statement

Here are some equations that contain a **variable** and a list of values. Think about what each equation means and find a **solution** in the list of values. If you get stuck, draw a diagram. Be prepared to explain why your solution is correct.

1. $1000 - a = 400$
2. $12.6 = b + 4.1$
3. $8c = 8$
4. $\frac{2}{3} \cdot d = \frac{10}{9}$
5. $10e = 1$
6. $10 = 0.5f$
7. $0.99 = 1 - g$
8. $h + \frac{3}{7} = 1$

Values:

- $\frac{1}{8}$
- $\frac{3}{7}$
- $\frac{4}{7}$
- $\frac{3}{5}$
- $\frac{5}{3}$
- $\frac{7}{3}$
- 0.01
- 0.1
- 0.5
- 1
- 2
- 8.5
- 9.5
- 16.7
- 20
- 400
- 600
- 1400

Possible Responses

1. 600
2. 8.5
3. 1
4. $\frac{5}{3}$
5. 0.1
6. 20
7. 0.01
8. $\frac{4}{7}$

Anticipated Misconceptions

Instead of solving, students might follow the operation symbol and combine the numbers in that way (for example, adding 12.6 and 4.1 to get 16.7 for the equation $12.6 = b + 4.1$). Encourage students to express the relationships of the equation in words and to draw diagrams that describe those statements. $12.6 = b + 4.1$ can be stated as “When a number is added to 4.1, the sum is 12.6.” The tape diagram then shows the parts 4.1 and an unknown quantity b, and a total of 12.6.

Are you ready for more?

One solution to the equation $a + b + c = 10$ is $a = 2, b = 5, c = 3$.

How many different whole-number solutions are there to the equation $a + b + c = 10$? Explain or show your reasoning.

"Are you ready for more?" Student Response

If a, b, and c are *positive*, there are 36 solutions. If $a = 1$, the possibilities are that $b = 1$ and $c = 8$, $b = 2$ and $c = 7$, and so on, giving 8 solutions. If $a = 2$, then b and c could respectively be 1 and 7, 2 and 6, 3 and 5, etc. This gives 7 solutions for $a = 2$. If $a = 9$ and all numbers are positive, there are no possible numbers for both b and c. The total number of solutions (for a value of 1 through 8) is $8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$ or 36.

If a, b, and c *non-negative* and includes 0, there are 66 solutions. If $a = 0$, there are 11 combinations of b and c. If $a = 1$, there are 10 combinations, and so on. The total number of solutions (for a value of 0 through 10) is $11 + 10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1$, which is 66.

If a, b, and c are *any* integers (including negative), then there is an unlimited number of solutions.

Lesson Summary

Review understanding and appropriate use of new vocabulary (variable, coefficient, solution) and the concepts they represent. What does it mean for an equation to be true? False? Is an equation with a variable always true?

2.4 How Do You Know a Solution is a Solution?

Cool-down: 5 minutes

Student-Facing Task Statement

Explain how you know that 88 is a solution to the equation $\frac{1}{8}x = 11$ by completing the sentences:

The word "solution" means . . .

88 is a solution to $\frac{1}{8}x = 11$ because . . .

Possible Responses

The word "solution" means a value that makes the equation true.

88 is a solution to $\frac{1}{8}x = 11$, because if x is 88, the equation is $\frac{1}{8} \boldsymbol{\cdot} 88 = 11$, which is true.